Alternating Current

Objectives

After going through this lesson, the learners will be able to:

- Understand the design principle and working of an ac generator
- Know the meaning of the term alternating current and relate it to its origin
- Differentiate alternating current from direct current
- Appreciate the reason for considering 'sinusoidal time variation' as the basic or fundamental form of alternating current
- Draw and represent AC and DC waveforms
- Recognize use of AC or DC, for household and commercial applications
- Distinguish between the instantaneous value, the peak value and the *rms* value of an alternating current
- Know the physical significance of the *rms* value of an alternating current or alternating voltage
- Define the *rms* value of an alternating voltage and obtain the relation between this value and the peak value of the alternating voltage
- Conceptualize the use of phasors for AC circuits

Content Outline

- Unit Syllabus
- Module Wise Distribution of Unit Syllabus
- Words You Must Know
- Introduction
- Generating Direct Current or DC Sources
- Generating AC
- AC Generator
- Using GeoGebra to Visualize AC
- Phasors
- Advantage of AC over DC
- Expressions for Alternating Current and Alternating Voltage
- Average and Mean Value of Alternating Current and Voltage

- Root Mean Square Value of Alternating Current and Voltage
- Summary

Unit Syllabus

Unit IV: Electromagnetic Induction and Alternating Currents

Chapter-6: Electromagnetic Induction

Electromagnetic induction; Faraday's laws, induced emf and current; Lenz's Law, Eddy currents; Self and mutual induction.

Chapter-7: Alternating Current

Alternating currents, peak and rms value of alternating current/voltage; reactance and impedance; LC oscillations (qualitative treatment only), LCR series circuit, resonance; power in AC circuits, wattless current; AC generator and transformer.

Module Wise Distribution of Unit Syllabus 09 Modules

The above unit is divided into 9 modules for better understanding.

Module 1	• Electromagnetic induction • Faraday's laws, induced emf and current • Change of flux • Rate of change of flux
Module 2	• Lenz's Law • Conservation of energy • Motional emf
Module 3	• Eddy currents • Self induction • Mutual induction • Unit • Numerical
Module 4	• AC generator • Alternating currents • Representing ac • *Formula* • Graph

	• Phasor • Frequency of ac and what does it depend upon • peak and rms value of alternating current/voltage
Module 5	• ac circuits • Components in ac circuits • Comparison of circuit component in ac circuit with that if used in dc circuit • Reactance mathematically • Pure R • Pure L • Pure C • Phasor, graphs for each
Module 6	• AC circuits with RL, RC and LC components • Impedance; LC oscillations (qualitative treatment only) • Resonance • Quality factor
Module 7	• Alternating voltage applied to series LCR circuit • Impedance in LCR circuit • Phasor diagram • Resonance • Power in ac circuit • Power factor • Wattles current
Module 8	• Transformer
Module 9	• Advantages of ac over dc • Distribution of electricity to your home

Module 4

Words You Must Know

Let us remember the words we have been using in our study of this physics course:

- **Magnetic Field:** The region around a magnet, within which its influence can be felt.
- **Electromotive Force:** The amount of work done by an external source, to take a unit positive charge once round the circuit.

- **Area Vector:** A vector perpendicular to a given area whose magnitude is equal to the given area.
- **Electromagnetic Induction:** The phenomenon in which electric current can be generated by varying magnetic fields is called electromagnetic induction (EMI).
- **Magnetic Flux:** Just like electric flux, magnetic flux ϕ_B through any surface of area A held perpendicularly in magnetic field B is given by the total number of magnetic lines of force crossing the area. Mathematically, it is equal to the dot product of B and A.

  $\Phi_B = B. A = BA \cos \theta$, where θ is the angle between B and A
- **Induced emf and Induced Current:** The emf developed in a loop when the magnetic flux linked with it changes with time is called induced emf. When the conductor is in the form of a closed loop, the current induced in the loop is called an induced current.
- **Weber:** One weber is defined as the amount of magnetic flux, through an area of $1m^2$ held normal to a uniform magnetic field of one tesla. The SI unit of magnetic flux is weber (Wb) or tesla metre square (Tm^2).
- **Faraday's Laws of Electromagnetic Induction:**
  - **First Law**: It states that whenever the amount of magnetic flux linked with the coil changes with time, an emf is induced in the coil. The induced emf lasts in the coil only as long as the change in the magnetic flux continues.
  - **Second Law:** It states that the magnitude of the emf induced in the coil is directly proportional to the time rate of change of the magnetic flux linked with the coil.
- **Lenz's Law:** The law states that the direction of induced emf is always such that it opposes the change in magnetic flux responsible for its production.
- **Fleming's Right Hand rule:** Fleming's right hand rule gives us the direction of induced emf/current in a conductor moving in a magnetic field.
- If we stretch the fore-finger, central finger and thumb of our right hand mutually perpendicular to each other such that the fore-finger is in the direction of the field, the thumb is in the direction of motion of the conductor, then the central finger would give the direction of the induced current.
- **Induced emf by Changing the Magnetic Field:** The movement of magnet or pressing the key of coil results in changing the magnetic field associated with the coil, this induces the emf.

- **Induced emf by Changing the Orientation of Coil and Magnetic Field:** When the coil rotates in a magnetic field the angle Θ changes and magnetic flux linked with the coil changes and this induces the emf. This is the basis of ac generators.
- **Induced emf by Changing the Area A(Motional EMF):** Motional emf is a type of induced emf which occurs when a wire is pulled through the magnetic field. The magnitude of motional emf depends upon the velocity of the wire, strength of magnetic field and the length of the wire.

  Motional emf arises due to the motion of charges due to a magnetic field.

Introduction

The phenomenon of electromagnetic induction has been technologically used in many ways.
An exceptionally important application is the generation of alternating currents (ac). The modern ac generator with a typical output capacity of 100 MW is a highly evolved machine.
In this section, we shall describe the basic principles behind this machine. The Yugoslav inventor Nicola Tesla is credited with the development of the machine.

An ac generator converts mechanical energy into electrical energy.

One method to induce an emf or current in a loop is through a change in the loop's orientation or a change in its effective area. As the coil rotates in a magnetic field B, the effective area of the loop (the face perpendicular to the field) is $A \cos \theta$, where θ is the angle between A and B. This method of producing a flux change is the principle of operation of a simple ac generator.

Alternating current (AC) electricity is the electricity that is mostly used in homes and businesses throughout the world.

Direct Current (DC) electricity flows at a steady rate, in one direction through a wire; AC electricity alternates its direction in a back-and-forth way. The direction of flow (usually) alternates between 50 to 60 times per second, depending on its generating system.

AC electricity is produced by an AC electric generator, this determines its frequency.

In AC electricity, the 'voltage' can be readily changed; this makes it more suitable for long-distance transmission than DC electricity.

Further AC electricity makes ample and frequent use of capacitors and inductors in electronic circuitry, allowing for a wide range of applications. AC electricity often referred to as just as AC, which is also the abbreviation for air conditioning. It is better therefore, to use the full name: AC Electricity.

Current is defined as the rate of flow of electric charge through a given cross-section.

Depending on the way of the flow of electric charge, the current can be categorized into:

(i) Alternating Current (AC)

(ii) Direct Current (DC).

Direct Current Sources provide DC; that is currents that do not change their direction with time.

When the direction of the flow of electric charge changes its direction, with time in a periodical way, it is called an Alternating Current.

Alternating Currents are used for efficient transmission and distribution of electrical energy.

Source image: wikkipedia.org

Difference between AC and DC electricity

This table shows the points of differences between Alternating Current and Direct current we work with both in our daily life.

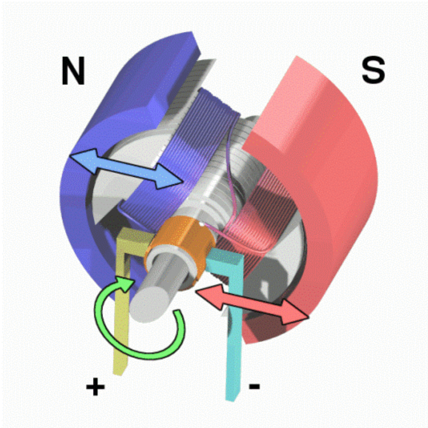
Points of difference	**Alternating Current**	**Direct Current**
Amount Of Energy that Can be transmitted	Can be transferred over longer distances with very much reduced 'transmission' related energy losses.	DC transmission, over long distances, will cause high 'energy losses' during transmission.

Cause of the direction of flow of electrons	Change in magnetic flux linked with a rotating coil.	A steady 'electric field' along the wire.
Frequency	The frequency of the transmitted alternating current is usually 50Hz or 60Hz depending upon the country.	The frequency of direct current is zero.
Direction	It keeps on reversing its direction, periodically while flowing in a circuit.	It flows in one direction in the circuit. conventionally from high to a low potential
Current	The magnitude of current keeps on varying with time.	It is usually a current of constant magnitude, in a given circuital set-up.
Flow of Electrons	Electrons keep switching their direction of motion forward and backward	Electrons 'drift' move steadily in one direction 'forward', only
Obtained from	A.C Generators which 'feed' the mains.	Cell or Battery
Passive Parameters	Impedance	Resistance only
Power Factor	It lies between 0 & 1	It is always 1
Types	Sinusoidal, Trapezoidal, Triangular, Square.	Steady; Uni-directional.

The differences and similarities will become clear as you understand alternating current better.

Generating Direct Current or DC Sources

We have chemical cells and batteries, solar cells, rectifiers and adapters used in mobile chargers and also DC generators which work on electromagnetic induction and special machine designs .watch Arvind Gupta toys link to make your own DC generator.



Source image: creative commons,wikipedia.org

https://www.youtube.com/watch?v=vSPFwibREUg

Generating Alternating Current

According to Faraday's Law of Induction, a changing magnetic flux can induce an emf.

The methods to generate induced emf are methods of changing flux $\phi = B\,A\cos\theta$.

a. Changing the magnetic field B
b. Changing the area A of the coil
c. Changing the relative orientation θ of B and A

If a coil rotates with uniform angular velocity in a uniform magnetic field, the induced emf varies sinusoidally with time; this can lead to an alternating current. If the coil is connected to a resistance and makes a closed circuit allowing alternating current to flow in the circuit, this works as a generator of Alternating current . The following symbol is used to show the AC Voltage source.

Represented by

Source image: wikkipedia.org

The unit of induced emf is volt

AC sources: AC Generator

You have learnt about electricity generation at a thermal power station, or a hydro electric power station or perhaps, even a nuclear power station.

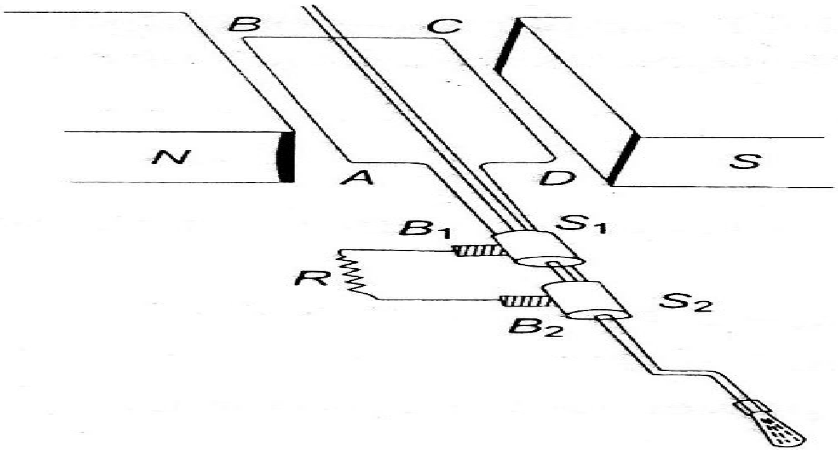
How does the energy of the steam in thermal power, of the falling water in hydroelectric power or of superheated steam generated by the nuclear reactor get utilized to generate alternating emf?

This energy is used to rotate a turbine, which in turn rotates the armature coil (of an AC generator) put in a magnetic field.

An A.C. dynamo/generator is a device that generates an alternating current using mechanical energy (i.e. converts mechanical energy into electrical energy of A.C.).

Principle:

The principle of a generator working is **electromagnetic induction**, i.e. whenever the magnetic flux linked with a coil changes, an *e.m.f.* is induced in the coil which lasts only as long as the magnetic flux changes. The direction of the induced alternating current is given by Fleming's Right Hand Rule following Lenz's law.



Construction:

A schematic diagram of the ac generator is shown above.

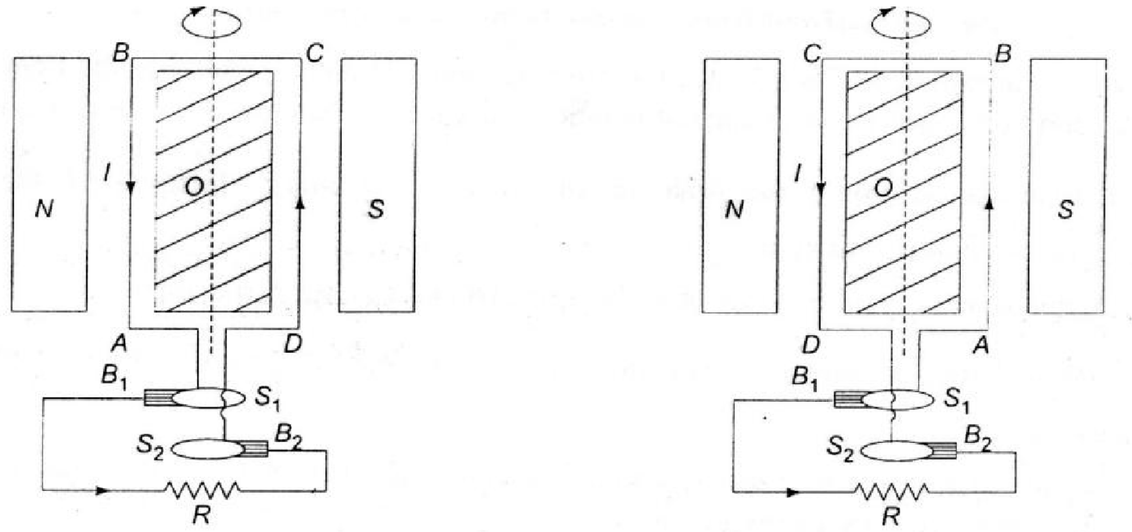
The essential parts are:

- **An Armature:** ABCD, which is a rectangular insulated copper wire coil of several turns. The coil can be rotated about a central axis perpendicular to a uniform magnetic field.
- **A soft Iron Laminated Core:** over which the armature coil is wound, is used to increase the magnetic flux linked with the coil. This core is laminated to reduce eddy currents.
- **Magnetic Poles:** N and S of a strong electromagnet, between which the armature is rotated (about an axis perpendicular to the magnetic field lines).
- **Slip Rings:** S_1 and S_2, are two hollow metallic rings to which the two ends of the armature coil are connected (soldered) and which rotate along with the coil.
- **Carbon Brushes:** B_1 and B_2 (kept fixed) make touch contact with the surfaces of S_1 and S_2 respectively, and carry the current from the armature coil to the circuit load resistance R.
- **R is the Load Resistance:** in the external circuit, across which the output A.C. is obtained.

Working:

- The armature coil is rotated as shown, about an axis perpendicular to the magnetic field lines. As the angle between the area vector of the coil and the magnetic field changes (see Fig.) the magnetic flux linked with the coil changes and an e.m.f. is induced in the coil.

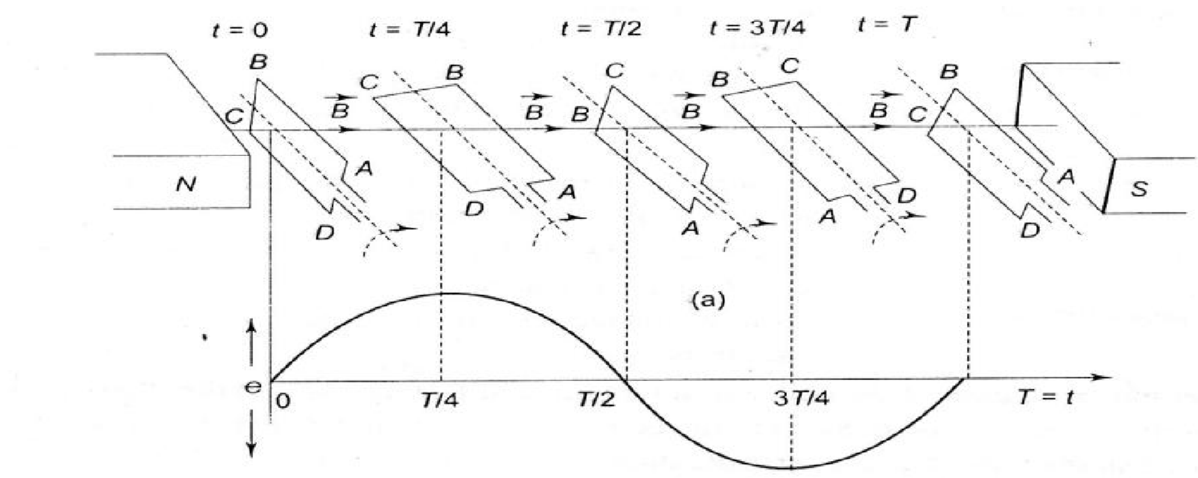
- When the plane of the coil is parallel to the plane of the paper, the flux linked with the coil is zero. As the coil is rotated as shown in fig. (i), AB moves in and CD moves out of the screen, causing current I to be induced in the coil in the direction DCBAD, through the resistance R in the external circuit as shown.
- After half a rotation of the coil. AB moves out and CD moves into the plane of the screen, causing current I to be induced in the coil in the direction ABCDA, through resistance R in the external circuit in the opposite direction as in fig. (ii).



Direction of induced current changes after half a cycle of rotation of the coil.

The working of an A.C generator may be explained with the help of five different positions of the armature coil ABCD at time t = 0,T/4,T/2,3T/4 and t =T .

Of course this is assuming that the coil is rotated in uniform magnetic field B with constant angular velocity ω .$where\ \omega = \frac{2\pi}{T}$ **. T is the periodic time or time for one complete rotation of the coil.**



Notice the change in direction of induced current due to induced emf. The graph of e vs time shows the variation in voltage as a consequence of rotation of the coil. Also see when we get maximum emf and when its value becomes zero.

For one complete rotation there are two instants when emf is max at T/4 and at 3T/4.

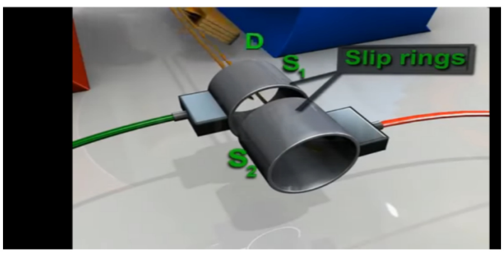
The same would be repeated for the next rotation

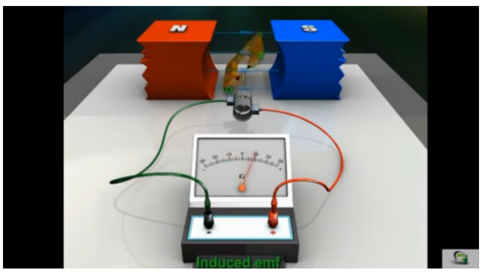
Think About This

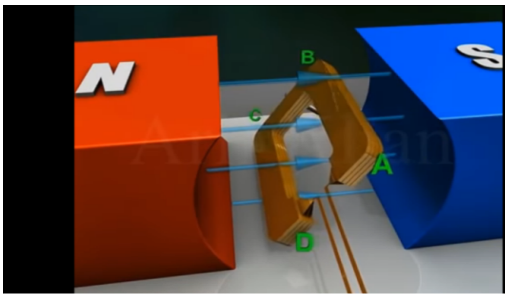
- **What if angular velocity ω is not constant?**
- **What if the magnetic field is not uniform?**
- **What if the axis of rotation is not fixed?**
- **What if the coil is wound on a solid cylinder?**
- **What if the coil winding is in a plane along B?**

Watch the following video

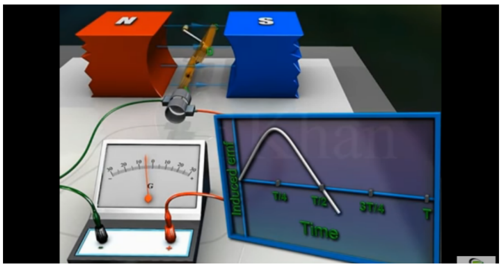
https://www.youtube.com/watch?v=7_VvluGM66c

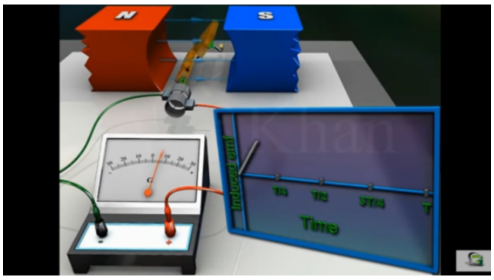






.

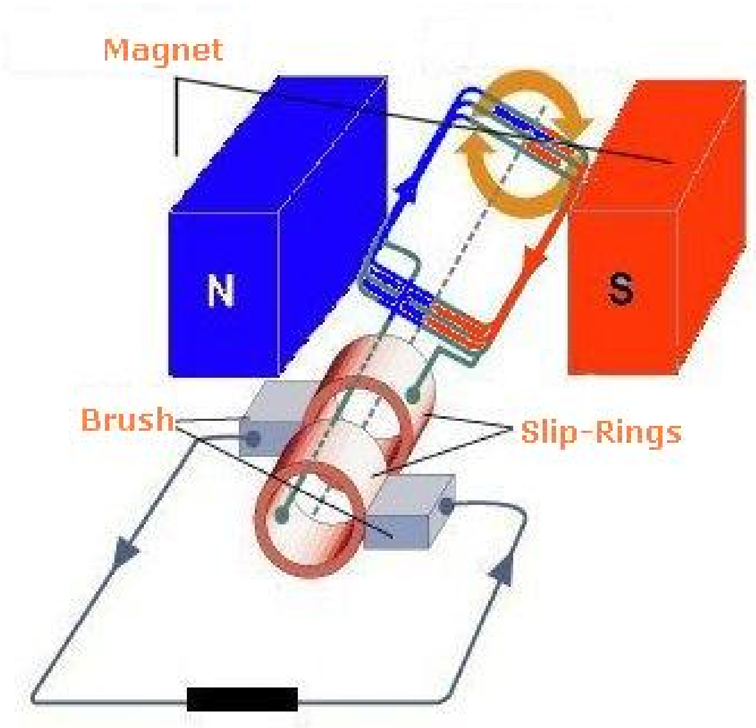




Alternating voltage may be generated by rotating a coil in the magnetic field.

The value of the voltage generated depends on-

i. The number of turns in the coil. N
ii. Strength of the field. B
iii. The speed ω at which the coil or magnetic field rotates



Source:http://www.ncert.nic.in/html/learning_basket/electricity/electricity/machine/ac_generator.htm

Calculating Induced Emf

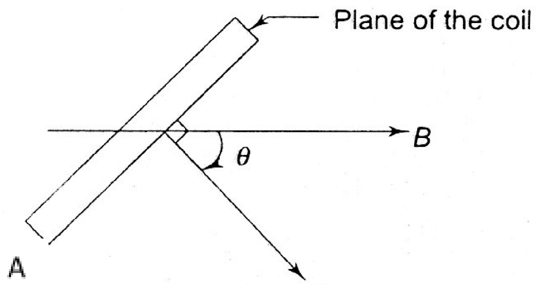
Consider a rectangular coil having N turns and rotating in a uniform magnetic field B (directed along the z-axis, say) with an angular velocity of ω radian/second.

Maximum flux ϕ_B is linked with the coil when its plane coincides with the x-axis.

At instant time t seconds, this coil rotates through an angle $\theta = \omega t$.

So

When the coil is rotated with a constant angular speed ω, the angle θ between the magnetic field vector B and the area vector A of the coil at any instant *t* is $\theta = \omega t$. (assuming $\theta = 0^o$ at $t = 0$).



Angle θ changes with time

As a result, the effective area of the coil exposed to the magnetic field lines changes with time, the flux at any time *t* is

$$\phi_B = BA\cos\theta = BA\cos\omega t$$

From Faraday's law, the induced emf for the rotating coil of *N* turns is:

$$e = -N\frac{d\phi_B}{dt} = NBA\frac{d}{dt}\cos\omega t$$

Thus, the **instantaneous value of the emf** is:

$$e = N\,B\,A\,\omega\,\sin\omega\,t$$

Where N B A ω is the maximum value of emf, which occurs when sin ωt is +1 or -1

Points to note

- We say instantaneous induced emf, but at an instant the flux around the coil is fixed so Faraday's law should not apply. Think about instantaneous velocity where the object changes position in a small interval of time, the interval tending to be zero - but not zero .
- The rate of change of flux depends upon $\frac{d}{dt}\cos\omega t$ if N A and B are constant

The differentiation $\frac{d}{dt} cos\omega t$ gives ω sin ω t

- The induced emf e = N B A ω sin ω t depends upon
  - i) N
  - ii) B
  - iii) A and
  - iv) ω
- Since the value of the sine function varies between +1 and –1, the sign, or polarity of the emf changes with time.
- The emf has its maximum value when θ = 90º or θ = 270º, as the change of flux is greatest at these points.The maximum is $\pm NBA\omega$
- The magnitude of alternating current changes continuously with time
- The **direction of the current changes periodically and therefore the current is called *alternating current*** (ac).
- Emf is induced in the coil whether the coil is rotated clockwise or anticlockwise
- The induced emf in the coil is zero when its plane is normal to the magnetic field even though maximum magnetic flux is linked with the coil in this position
- Alternating current can be represented mathematically using sinusoidal functions e.g. sine and cosine functions.
- Alternating current can be represented mathematically using graphs voltage – time or current –time.

Example:

The coil of an ac generator has a frequency of 50 Hz and the maximum voltage developed is 310 V. Area of the coil is 3 x 10 $^{-3}$ m^2 and the number of turns in the coil is 500. Find the strength of the magnetic field in which the coil rotates.

Solution:

$$B = \frac{e}{NA\omega} = \frac{310}{500\times3\times10^{-3}\times2\times3.14\times50}$$

=0.658 T

Example:

Kamla peddles a stationary bicycle; the pedals of the bicycle are attached to a 100 turn coil of area 0.10 m^2. The coil rotates at half a revolution per second and it is placed in a uniform magnetic field of 0.01 T perpendicular to the axis of rotation of the coil. What is the maximum voltage generated in the coil?

Solution:

Here $f = 0.5$ Hz;

$N = 100$,

$A = 0.1\ m^2$

and $B = 0.01$ T

$e = N A B\,(2\pi f)$

$e = 100 \times 0.01 \times 0.1 \times 2 \times 3.14 \times 0.5$

$= 0.314$ V

Example:

A bicyclist is traveling on a straight dark road. At what speed should he travel to generate electricity to light a 6 V bulb. If the coil of the generator has 75 turns, area of $3 \times 10^{-3} m^2$ radius of cycle wheel = 0.33 m. The angular velocity of the rotating coil is made 44 times that of the tyre by using suitable gears.

Solution:

$$\omega = \frac{e}{NAB}$$

$$V = \omega r = \frac{e}{NAB} r\left(\frac{1}{44}\right) = \frac{6 \times 0.33}{75 \times 3 \times 10^{-3} \times 0.1 \times 44}$$

$= 2\ ms^{-1}$

or 7.2 km/h

Using Geogebra to Visualize AC

GeoGebra is the graphing calculator for functions, geometry, algebra, calculus, statistics and 3 D mathOr it is a dynamic tool for mathematics learning and teaching

Now if you have used it before you will understand how this is a powerful tool based on mathematical graphing to see the sine or cosine function

To enjoy the GeoGebra app you must first download it.

A Geogebra Tutorial

The idea behind this tutorial is to introduce you quickly to the major capabilities of GeoGebra, and give you the tools to explore the details on your own. As much as possible, try to move from basic to more advanced, emphasizing the features most commonly used.

You have to download GeoGebra from www.geogebra.org and have it set up on your computer, As you go through these tutorials, feel free to experiment.

Geogebra and AC

Instantaneous value of the induced emf is:

$e = N\,B\,A\,\omega\, sin\,\omega\, t$

Where N B A ω is the maximum value of emf, which occurs when sin ω t is +1 or -1

If you look at the equation carefully:

N (number of turns of the coil) B (the magnitude of the external magnetic field). A (area of the coil face) ω (angular velocity of the rotating coil) are all constants but the value of sin ω t changes with time t so 0 -T

T being the periodic time so $\omega\, t = \frac{2\pi}{T} t = 2\pi f t$

Notice: here sin ω t may be said to depend on instant 't' as T (time period or time for one rotation) and f (frequency of rotation of coil or number of times the coil rotates about its axis per second)

ω is called angular frequency = $2\pi f$**, note angular frequency has no physical existence but a mathematical result as a product of** $2\pi f$ **its unit is rad** s^{-1}**.**

Therefore geogebra can be used to show the induced emf 'e'.

Phasor

"A vector that represents a sinusoidally varying quantity, as a current or voltage, by means of a line rotating about a point in a plane, the magnitude of the quantity being proportional to the length of the line and the phase of the quantity being equal to the angle between the line and a reference line"

In the above definition the word vector is used, which we know are representative lies with arrows used to indicate physical quantities that have both magnitude and direction.

Phasors are not vectors in that sense.

- A phasor that represents a sinusoidally varying quantity
- A phasor can be imagined to rotate anticlockwise with angular velocity equal to angular frequency.
- Length of the phasor represents the magnitude of the physical quantity.
- The projection upon reference axis gives the instantaneous value of the quantity.
- The phase angle between two phasors represents the phase difference between two physical quantities represented by the phasor.

- Study of alternating currents and voltages is greatly simplified if phasors are used to represent the two physical quantities.
- Phasor diagram represents alternating current and voltage of the same frequency as rotating vectors (phasors) along with proper phase relation between them.
- **The links given show periodically changing sine wave the blue dot may represent magnitude of alternating current or voltage**
  Source image: wikkipedia.org
  https://upload.wi.org/wikipedia/commons/4/41/AC_wave_Positive_direction.gif
- **Geogebra tool showing sine wave, and phasor along with it . you may change the frequency or magnitude to see the changes graphically. these help us imagine alternating currents and voltages in circuits**
  Sourcelink-
  https://www.geogebra.org/m/jx9fpmUp?doneurl=%2Fsearch%2Fperform%2Fsearch%2Fsine%2Bwave%2Bphasor%2Bvalue%2Fmaterials%2F

Phasor Diagram

Phasor diagrams are diagrams representing alternating current and voltage of the same frequency, as 'vectors' or 'phasors' with the 'phase angle' between them. Phasors are the arrows rotating in the anti-clockwise direction i.e. they are rotating vectors but they however, represents scalar quantities.

Thus a sinusoidal alternating current and voltage can be represented by an anti-clockwise rotating vector through the following conditions.

The length of the vector is taken as equal to the peak value of alternating voltage or current.

Vectors representing alternating current and voltage would be at a horizontal position, at the instant when alternating quantity is zero. In certain circuits the current reaches its maximum value after the emf becomes maximum.

The current is then said to lag behind emf. When current reaches its maximum value before emf reaches its maximum value the current is then said to lead the emf.

Figure below shows the current lagging behind the emf by 90^0.

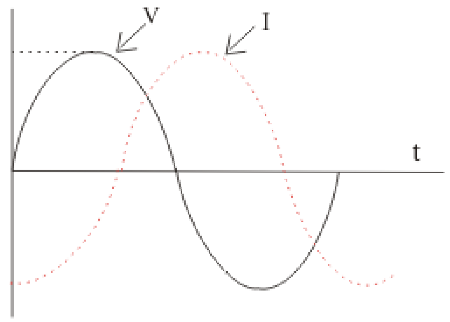


Figure 2 (a). Sinusoidal representation

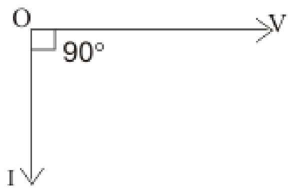


Figure 2 (b). Phaser diagram representation where vector OV is peak value of voltage and OI is peak value of current

SINE WAVE USING GEOGEBRA.ggb

https://www.geogebra.org/m/UPughP3s

The current will oscillate with the same frequency as the voltage source, with an amplitude I_0 and phase φ that depends on the driving frequency and the circuit used.

When the electrons in the alternating current flow or switch their direction, the direction of current and the voltage of the circuit reverse itself.

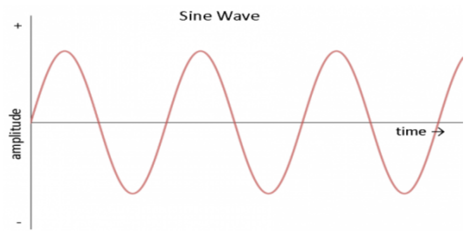
In public power distribution systems in India, (including household current), the voltage reverses itself 50 times per second. In some countries, like the United States the voltage reverses itself 60 times per second. The rate at which alternating current reverses direction is called its **frequency**, this is expressed in hertz. Thus, the standard household current in India is a 50 Hz AC.

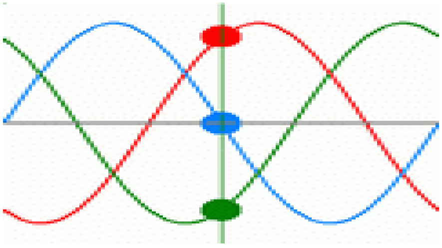
In commercial generators, the mechanical energy required for rotation of the armature is provided by water falling from a height, for example, from dams. These are called **hydro-electric generators.** Alternatively, water is heated to produce steam using coal or other sources. The steam at high pressure produces the rotation of the armature. These are called **thermal generators.** Instead of coal, if a nuclear fuel is used, we get **nuclear power generators**. Modern day generators produce electric power as high as 500 MW, i.e., one can light up 5 million 100 W bulbs!

In most generators, the coils are held stationary and it is the electromagnets which are rotated. The frequency of rotation is 50 Hz in India.

Waveforms for Alternating Current

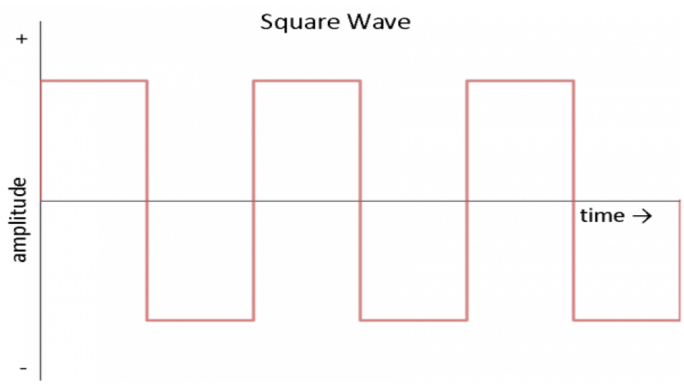
There can be various waveforms for AC; the only requirement is to have regularly periodical alternating voltage and current. Usually, AC varies according to a sinusoidal wave function. Some of the examples of AC waveforms are shown below:





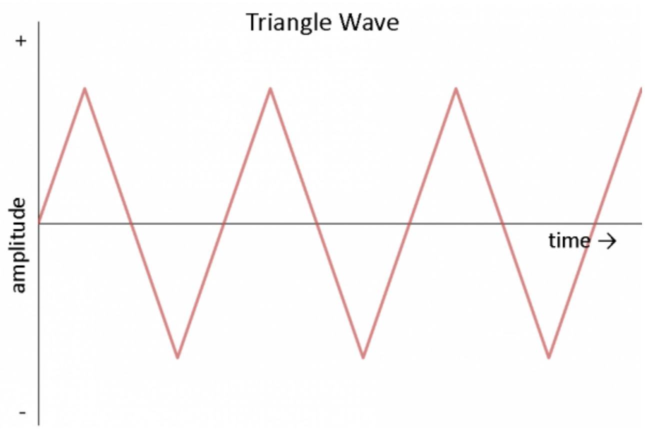
Source image: wikkipedia.org

Similar to sine waves, we can also have square waves and triangle waves for Alternating Current.



Source image: wikkipedia.org

Square waves are used in digital and switching electronics.



Source image: wikkipedia.org

This link shows a sine wave, square or triangular wave using a slider with an oscilloscope, a device to graphically see the wave on a screen.

http://www.physics-chemistry-interactive-flash-animation.com/electricity_electromagnetism_interactive/oscilloscope_V_T.htm

Triangle waves are found in sound synthesis and are useful in linear electronics like amplifiers.

Advantages of AC over DC Electricity

The major advantage that AC electricity has over DC electricity is that AC voltages can be readily transformed to higher or lower voltage levels, while it is difficult to do that with DC voltages. Further the high voltages, from the power station, can be easily reduced to a safer voltage for use in the house. It is done by the use of a transformer. It is easier to obtain a large value of AC.

The advantage can be best understood on completion of your study of transformers

Expression for Alternating Current and Alternating Voltage

Usually, an alternating current is one whose magnitude changes sinusoidally with time .Thus alternating current is given by:

$e = N\,B\,A\,\omega\, sin\,\omega\, t$

Here N B A ω is called **maximum value or peak value** of induced emf

If the external circuit resistance is R

Induce current can be given by:

$I = e/R$

$i = i_0\, sin(\omega t + \Phi)$.......(1)

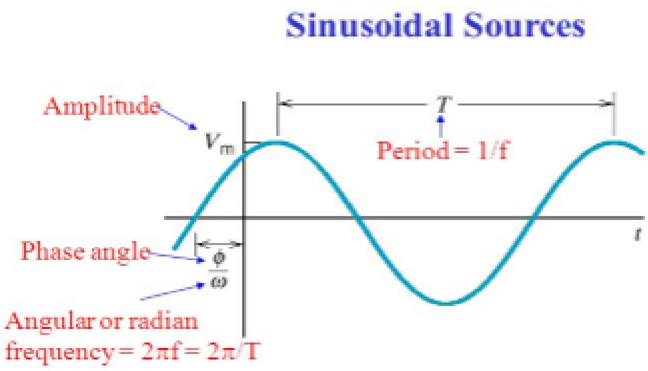
here,

i_0= **current amplitude or peak value** of alternating current.

If T is the time period of alternating current and f is the frequency, then

$\omega = \frac{2\pi}{T} = 2\pi f$...........(2)

Here, ω is called **angular frequency** of A.C, φ is known as the **initial phase constant**.



- Instead of sine function AC can also be represented by the cosine function and both representations lead to same results. We will discuss circuits with sine representation of A.C
- The given figure shows the **sinusoidal variation of A.C with time.**

  A complete set of variations of the current over one time period T is called a **cycle.**
- The emf or voltage whose magnitude changes sinusoidally with time is known as a (sinusoidal) alternating emf; it is represented by:

  $V = V_0 \sin(\omega t + \Phi)$.....(3)

  where V_0 is the peak value of the alternating voltage.

Average or Mean value of Current and Voltage

If an alternating current were passed through a moving coil galvanometer it would show no net deflection, this is because for one complete cycle, **the mean or average value of alternating current is zero.** The AC flows in one direction during one half cycle and in the opposite direction during the other half cycle. **The mean value of A.C, over a half cycle is not zero.**

The mean or average value of AC, can be defined using either its positive half cycle or its negative half cycle.

We have,

$$i_{avg(\frac{T}{2})} = \frac{\int_0^{T/2} i\,dt}{\int_0^{T/2} dt} = \frac{\int_0^{T/2} i_0 \sin(\omega t+\Phi)}{\int_0^{T/2} dt} = \frac{2i_0}{\pi} \cong 0.636\, i_0 .... (4)$$

From equation (4), **we see that the average value of A.C during a half cycle is 0.636 times or 63.6% of its peak value**

Similarly we can show that:

$$V_{avg(T/2)} = \frac{2V_0}{\pi} \cong 0.636\, V_0$$

During the next half cycle, the mean value of ac will be equal in magnitude but opposite in direction.

Always remember that the mean value of AC over a complete cycle is zero, it can be defined only over a half cycle of AC.

Root Mean Square Value of Alternating Current and Voltage

We know that the time average value of AC over one cycle is zero. The instantaneous current, I and the time average of AC over a half cycle would be positive for one half cycle and negative for the other half cycle;

However, the quantity i^2 would always remain positive. The time average of quantity i^2 would be given by:

$$\bar{i}^2 = \frac{\int_0^T i^2 dt}{\int_0^T dt}$$

$$= \frac{1}{T}\int_0^T i_0^{\ 2} sin(\omega t + \Phi)dt$$

$$= \frac{i_0^{\ 2}}{2T}\int_0^T [1 - cos(\omega t + \Phi)\,]dt$$

$$= \frac{i_0^{\ 2}}{2T}\left[t - \frac{sin^2(\omega t+\Phi)}{2\omega}\right]_0^T$$

$$= \frac{i_0^{\ 2}}{2T}\left[T - \frac{\sin(4\pi+2\Phi) - sin2\Phi}{2\omega}\right]$$

$$= \frac{i_0^{\ 2}}{2} \qquad .... \qquad (5)$$

This is known as the **mean square current.**

The square root of mean square current is called root mean square current or rms current.

Thus,

$$i_{rms} = \sqrt{\bar{i}^2} = \frac{i_0}{\sqrt{2}} = 0.707\, i_0 \qquad ....(6)$$

Thus, the rms value of AC is 0.707 or 70.7% of the peak value of alternating current.

Similarly, rms value of alternating voltage or emf is:

$V_{rms} = \frac{V_0}{\sqrt{2}}$ ....(7)

Let the AC current, represented by I = $i_0$sin (ωt +φ), be made to pass through a resistor of resistance R.

The instantaneous power dissipated, due to the flow of current would be:

$P = i^2 R.$

Since magnitude of current changes with time, this power dissipation, in the circuit also changes with time.

The average power dissipated, over one complete current cycle would be:

$\overline{P} = \bar{i}^2 R = \left(i_{rms}\right)^2 R$

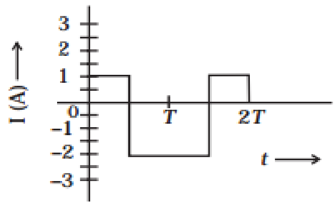
If we pass a direct current of (constant) magnitude i_{rms} through the same resistor, the power dissipated or rate of production of heat, in this case would be:

$P = (i_{rms})^2 R$

Thus, rms value of AC is that value of steady current which would dissipate the same amount of power in a given resistance in a given time as would have been dissipated by the alternating current.

This is why the rms value of AC is also known as the virtual value of current.

You can use the simulation to see the changes in peak value rms value or frequency
http://www.physics-chemistry-interactive-flash-animation.com/electricity_electromagnetism_interactive/sine_waveform_voltage_AC.htm



Do it yourself:

Short Answer Type

(i) Can the instantaneous power output of an AC source ever be negative? Can the average power output be negative?

(ii) The alternating current in a circuit is described by the graph shown in Fig. Show rms current from this graph.

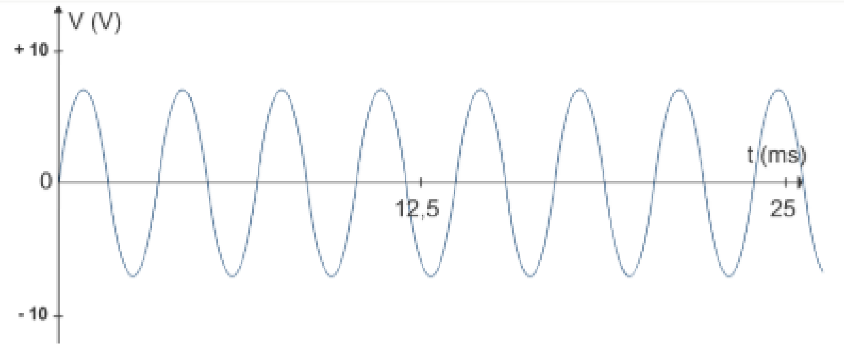
(iii) Both alternating current and direct current are measured in amperes. But how is the ampere defined for an alternating current?

Numerical

A 100 Ω resistor is connected to a 220V, 50Hz supply. Calculate rms value of current and net power consumed over a full cycle.

i. The peak voltage of an AC supply is 300V and rms value of current is 10A. Calculate the rms voltage and the peak current.

ii.



Find the peak value for the ac source having frequency of 60Hz, 220V.

Summary

- **Electric Current:** An electric current equals the rate of flow of electric charge. In electric circuits this charge is often carried by moving electrons in a wire. It can also be carried by ions in an electrolyte, or by both ions and electrons such as in plasma.
- **Voltage:** Voltage drop, electric potential difference, (electric pressure or electric tension) formally denoted by ΔV or ΔU, but more often simply as V or U, (for instance in the context of Ohm's or Kirchhoff's laws) is the difference in electric potential energy between two points per unit electric charge.
- **Induced Currents:** Electrical current induced within conductors by a changing magnetic field around it. They can be induced within (nearby) stationary conductors by a time-varying magnetic field.
- **Electric Generator:** An electric generator is an electrical machine that converts mechanical energy into electrical energy using the principle of electromagnetic induction.
- In an ac generator, mechanical energy is converted to electrical energy by virtue of electromagnetic induction. If coil of *N* turn and area *A* is rotated at ν revolutions per second in a uniform magnetic field *B*, then the motional emf produced is:

$\varepsilon = NBA\,(2\pi\nu)\,sin\,(2\pi\nu t)$

where we have assumed that at time t = 0 s, the coil is perpendicular to the field

- Alternating currents and voltages can be represented by sinusoidal graphs or phasors.
- **Phasors:** In Physics and Engineering, a phasor, is a complex number representing a sinusoidal function whose amplitude (A), angular frequency (ω), and initial phase (θ) are time-invariant. Basically, Phasors are rotating vectors.
- Alternating currents and voltages have **instantaneous value** given by:

$i = i_0\, sin(\omega t + \Phi)$

$V = V_0\, sin(\omega t + \Phi)$

- Alternating currents and voltages have **peak value I_0 and V_0**
- Alternating currents and voltages have **average values.**

$$i_{avg(\frac{T}{2})} = \frac{\int_0^{T/2} i dt}{\int_0^{T/2} dt} = \frac{\int_0^{T/2} i_0\, sin(\omega t+\Phi)}{\int_0^{T/2} dt} = \frac{2i_0}{\pi} \cong 0.636 i_0$$

$$V_{avg(T/2)} = \frac{2V_0}{\pi} \cong 0.636\, V_0$$

- Alternating currents and voltages have **root mean square values:**

$$i_{rms} = \sqrt{\bar{i}^2} = \frac{i_0}{\sqrt{2}} = 0.707\, i_0$$

$$V_{rms} = \frac{V_0}{\sqrt{2}}$$